import matplotlib.pyplot as plt

import math

# point cloud points sampled at equal intervals

# starts at wiki gate 24.5 and ordered by rising elevation

points29 = [[10000.4343159975, 10003.299618646875, 151.953300475], [10000.648772443887, 10003.263878160556, 151.95337676944445], [10000.755547534118, 10003.24106482706, 151.95335747294118], [10000.860968994, 10003.213721833334, 151.95348714200003], [10001.0694426975, 10003.150406258124, 151.953477860625], [10001.272739844284, 10003.074274965, 151.95347486214285], [10001.471592574375, 10002.984351875624, 151.953330993125], [10001.663917244166, 10002.881596345833, 151.9531987508333], [10001.8484595, 10002.76607077, 151.95332336349998], [10002.025077049231, 10002.639714370769, 151.95344074076922], [10002.193914353333, 10002.5009446725, 151.9535293575], [10002.352490536, 10002.352457332665, 151.95342814200004], [10002.501330042665, 10002.193259778667, 151.9533793126667], [10002.639783599998, 10002.024968451818, 151.95341075545454], [10002.766643714664, 10001.848434009999, 151.9534973146667], [10002.882039323751, 10001.663244188125, 151.953391075625], [10002.985170051536, 10001.471201448461, 151.9533198423077], [10003.075567090587, 10001.273687951763, 151.95344902470586], [10003.151998399999, 10001.069230361429, 151.953305925], [10003.21528022077, 10000.860869312308, 151.9533585776923], [10003.265189067502, 10000.649020885, 151.95350647], [10003.300922089093, 10000.434830996363, 151.95357444090908], [10003.302053403335, 9999.56463793, 151.95337219133336], [10003.077651341333, 9998.724781544668, 151.953297932], [10002.643296632858, 9997.971272315, 151.95345524285716], [10002.027632665331, 9997.356110335335, 151.95352376199997], [10001.274062068, 9996.921948656, 151.953385416], [10000.434342064376, 9996.698350936249, 151.953322411875], [9999.565196307502, 9996.699135287501, 151.953380585625], [9998.726016163528, 9996.923988917059, 151.95343286941178], [9997.973486998335, 9997.359220029166, 151.95330301833332], [9997.359712329997, 9997.974032782666, 151.95323282933333], [9996.924540043334, 9998.725334071332, 151.95345560666667], [9996.6985284975, 9999.564889060626, 151.95342922249998], [9996.697488896154, 10000.434261774617, 151.95350412307693], [9996.922709701537, 10001.273874786155, 151.953456], [9997.357637508125, 10002.026726916249, 151.95325756], [9997.972697972, 10002.641200271331, 151.95355835066667], [9998.72598900154, 10003.075448311538, 151.9533362753846], [9999.564940386923, 10003.300007233845, 151.95339613692306]]

points32 = [[10000.433985464, 10003.299608723999, 151.8771860753333], [10000.648612920768, 10003.264017306921, 151.87709397384617], [10000.755031306875, 10003.24092887375, 151.87733077937503], [10000.860467794735, 10003.213718639474, 151.87717558210525], [10001.069981286468, 10003.152074812942, 151.87710032705883], [10001.273573938824, 10003.074642364707, 151.87716046470587], [10001.471617510833, 10002.984985788331, 151.8771616616667], [10001.663829995556, 10002.882041427223, 151.87725830111114], [10001.848735146428, 10002.767098340713, 151.877192905], [10002.02534319846, 10002.640191077691, 151.8772219130769], [10002.19417621643, 10002.501750349998, 151.87707846499998], [10002.352559771429, 10002.353217959999, 151.8772539414286], [10002.502182586426, 10002.193831718572, 151.87719726714286], [10002.640534786875, 10002.02569699375, 151.87728500375], [10002.767445863332, 10001.848727177501, 151.87714767416665], [10002.882659238823, 10001.663974241765, 151.87724304117643], [10002.985355854617, 10001.471809231538, 151.87732755307695], [10003.075582872307, 10001.27379376154, 151.87709397538464], [10003.151582292858, 10001.069234072856, 151.87720053571428], [10003.215164834544, 10000.860995904544, 151.87729298000002], [10003.265023243686, 10000.648838525265, 151.87727355947368], [10003.30076047286, 10000.434131786431, 151.8770599364286], [10003.302192977144, 9999.564726642144, 151.8770948135714], [10003.077545153887, 9998.724628285, 151.87704976444445], [10002.642821073077, 9997.970696613846, 151.87723599999998], [10002.027710821665, 9997.355868736668, 151.8772574533333], [10001.273933791997, 9996.921901400667, 151.87709554133335], [10000.434125134001, 9996.698234860667, 151.87718505866667], [9999.565012058232, 9996.699294427646, 151.8773498511765], [9998.725375326669, 9996.922941127334, 151.877055866], [9997.972915257142, 9997.357673662143, 151.8770904542857], [9997.359668916155, 9997.973881024616, 151.87706345692305], [9996.924314425383, 9998.72548184923, 151.8770564153846], [9996.69831520375, 9999.56402731, 151.87703990874996], [9996.697291986251, 10000.43399219, 151.877134323125], [9996.92261011412, 10001.274136059414, 151.87721342058825], [9997.357192295385, 10002.026316515385, 151.87719609384615], [9997.972638181427, 10002.641138365, 151.8771166114286], [9998.725497274167, 10003.074852169166, 151.87727737416668], [9999.564967183751, 10003.30007654375, 151.877305031875]]

points35 = [[10000.434302724445, 10003.300356494445, 151.8010923622222], [10000.649284490624, 10003.264700397502, 151.80103111062502], [10000.755199776922, 10003.24173476154, 151.80103126], [10000.86044843143, 10003.214496425715, 151.8008444642857], [10001.069911856153, 10003.151830617693, 151.80093501153848], [10001.27337284077, 10003.074985302308, 151.80105943], [10001.472191944373, 10002.985002860001, 151.801088335], [10001.663975652667, 10002.882448402, 151.80104675333334], [10001.849105178333, 10002.767475943334, 151.801053365], [10002.026238038463, 10002.640250022307, 151.80088688692305], [10002.194613932308, 10002.501905716153, 151.8009150569231], [10002.353558184375, 10002.35364969125, 151.80091095], [10002.502680029285, 10002.194553578573, 151.8010929671429], [10002.641120181765, 10002.026060944705, 151.80097153529414], [10002.767742605884, 10001.849172817061, 151.80089434411764], [10002.883176875293, 10001.664096104118, 151.80088985529414], [10002.985741181818, 10001.471965085455, 151.80119185], [10003.076136883845, 10001.27370461231, 151.80090332], [10003.151793648236, 10001.069433296472, 151.80104693294115], [10003.215614540715, 10000.86025578643, 151.80105263857143], [10003.265125608666, 10000.649002667333, 151.80092468133336], [10003.301044855001, 10000.433675571428, 151.8008847914286], [10003.302398791539, 9999.564272846923, 151.80096787538463], [10003.077866335832, 9998.724458654999, 151.8009897866667], [10002.643569564001, 9997.970670495333, 151.801065064], [10002.0278723625, 9997.355390746665, 151.80087788749998], [10001.274575838463, 9996.921497766152, 151.8009901776923], [10000.433912167498, 9996.697456895834, 151.80115000416666], [9999.564959801333, 9996.698649627999, 151.80090535333335], [9998.724877270668, 9996.922121270001, 151.800963338], [9997.972148874165, 9997.3572781075, 151.80098978750001], [9997.359278038462, 9997.97346986154, 151.80107821], [9996.92363711, 9998.72546581909, 151.8009005454545], [9996.697475195, 9999.564137943125, 151.80092143937497], [9996.696613295333, 10000.434100267332, 151.8009338366667], [9996.92200011, 10001.2739384375, 151.800868034375], [9997.357185305002, 10002.02680450625, 151.801119805], [9997.972087292308, 10002.640925883845, 151.80081998307693], [9998.725239102727, 10003.07492405727, 151.80086170636363], [9999.564940396665, 10003.300413465999, 151.801071168]]

points38 = [[10000.433362230002, 10003.298773710001, 151.72485703461538], [10000.64869955889, 10003.263506610556, 151.72495354555554], [10000.754940643528, 10003.240896055882, 151.72471618647063], [10000.860167548668, 10003.213406928668, 151.7247680666667], [10001.068708368573, 10003.151115502144, 151.7248971114286], [10001.273069463077, 10003.074758126922, 151.7248582107692], [10001.470958872353, 10002.984263951765, 151.72482120411763], [10001.663435683333, 10002.881422806668, 151.72471008333335], [10001.848289588237, 10002.76660359647, 151.72480774058823], [10002.025382638749, 10002.639606298748, 151.72467803875003], [10002.192968805834, 10002.501254458335, 151.72486877583333], [10002.352082205998, 10002.352210601333, 151.72482299733332], [10002.50160191875, 10002.19363844375, 151.72486400625002], [10002.6395910375, 10002.025218546667, 151.72476959166664], [10002.766176402498, 10001.848297586668, 151.7248102841667], [10002.882376194999, 10001.663613523571, 151.72468457857144], [10002.984902533155, 10001.471333943684, 151.72483906], [10003.074558083335, 10001.273398207333, 151.7248179133333], [10003.1506202225, 10001.069337256877, 151.72483921125], [10003.214134472144, 10000.861300932142, 151.72468021857142], [10003.26411311375, 10000.648602131874, 151.72472286187502], [10003.299819689999, 10000.434228769229, 151.72478191692306], [10003.301377153335, 9999.564755332665, 151.72498474199998], [10003.077034278182, 9998.724729104548, 151.72486461363636], [10002.64272202077, 9997.971204756923, 151.72470797], [10002.027535548668, 9997.356181699999, 151.724842326], [10001.274378444285, 9996.922330208572, 151.72490365285717], [10000.433832889445, 9996.698608107778, 151.7248043488889], [9999.564837309375, 9996.699665904998, 151.724713325625], [9998.725154868667, 9996.923129273333, 151.72487182666666], [9997.972606316249, 9997.358089744375, 151.72494029937502], [9997.359925983528, 9997.974336709412, 151.7248992917647], [9996.924550368461, 9998.726089926922, 151.72478896076925], [9996.698406190588, 9999.564448115883, 151.72483825823525], [9996.697548443077, 10000.434108613077, 151.7248159546154], [9996.92296086143, 10001.273756010716, 151.72470746714285], [9997.357985236364, 10002.026119536364, 151.72492426272728], [9997.972463019336, 10002.640030813334, 151.72482503399996], [9998.725813619998, 10003.073771818748, 151.72476863875002], [9999.565126705882, 10003.299085980587, 151.7247134947059]]

points41 = [[10000.43313493941, 10003.296038235881, 151.6484545570588], [10000.647813998336, 10003.260646489998, 151.64846886611113], [10000.754050559375, 10003.237887815003, 151.648503303125], [10000.859723825883, 10003.210760481767, 151.64864573764706], [10001.068220082669, 10003.148846659335, 151.64882304], [10001.271465498572, 10003.071012412143, 151.64866093214283], [10001.46996437, 10002.981773941872, 151.64849090625], [10001.661863795, 10002.879659022143, 151.64855957000003], [10001.846487871251, 10002.764172509998, 151.64852237687504], [10002.023362079999, 10002.6370869875, 151.6486473075], [10002.191354224284, 10002.499246529287, 151.64871324785716], [10002.350425576667, 10002.350419394, 151.6486918133333], [10002.49935265091, 10002.191689338184, 151.64859008818186], [10002.637662224286, 10002.023663487142, 151.6485595692857], [10002.763977874616, 10001.846922213077, 151.64868046692305], [10002.878862996666, 10001.6619371775, 151.6486663816667], [10002.981868679333, 10001.469889457332, 151.64847615533333], [10003.071434123573, 10001.272256008571, 151.64871869857143], [10003.148013591335, 10001.06799534933, 151.64854431066667], [10003.21117403, 10000.860178162498, 151.648635865], [10003.260935672308, 10000.648454619231, 151.64866286076924], [10003.296968023335, 10000.433167252499, 151.64878718083332], [10003.298245237142, 9999.565021431426, 151.64857991666668], [10003.074157782145, 9998.725247579287, 151.64857264857145], [10002.640171576668, 9997.972538772667, 151.648617554], [10002.025420028667, 9997.357988294, 151.64867452], [10001.272136654285, 9996.924824970001, 151.6485225142857], [10000.433408435627, 9996.701615199376, 151.64847374], [9999.564786200002, 9996.702434046667, 151.64859822733334], [9998.726133843686, 9996.926636393157, 151.6486430910526], [9997.97406622357, 9997.360084959288, 151.64842442071432], [9997.362279256, 9997.975751590666, 151.6484415686667], [9996.927475162858, 9998.726615779287, 151.64874594642856], [9996.701260805625, 9999.564691751251, 151.648699759375], [9996.700222832858, 10000.433745144286, 151.64869253999998], [9996.925448134374, 10001.272546366878, 151.648674010625], [9997.360089708822, 10002.024206371763, 151.64847071058824], [9997.974491302306, 10002.637685666154, 151.64870159461537], [9998.726607918463, 10003.07076547769, 151.64849970846151], [9999.565000601335, 10003.296129337334, 151.64851481266663]]

points44 = [[10000.432357853571, 10003.291366167858, 151.57238006785718], [10000.64715615154, 10003.255980143846, 151.5723196176923], [10000.752938359286, 10003.23314622357, 151.5724280235714], [10000.85846762077, 10003.20660338077, 151.57248746538463], [10001.06622470143, 10003.143590519287, 151.57233974], [10001.269703065293, 10003.06733002353, 151.57253669352937], [10001.46747063294, 10002.977825755295, 151.57226652294116], [10001.659340333334, 10002.8750179625, 151.57242329916667], [10001.844065349165, 10002.760504085834, 151.57233556083335], [10002.020845028459, 10002.634305184616, 151.57249216], [10002.188822698, 10002.496167182666, 151.572388712], [10002.346903992666, 10002.347549136, 151.57251892000002], [10002.495608252, 10002.188839897333, 151.57233479799999], [10002.633192625, 10002.02048856857, 151.5724836092857], [10002.759959745334, 10001.844271867332, 151.57246093733337], [10002.874479440768, 10001.659744923078, 151.57261775076924], [10002.977248250001, 10001.467935175, 151.57242584083335], [10003.0672200725, 10001.26952674, 151.57243220083333], [10003.143670659289, 10001.066461494289, 151.57233864857145], [10003.20686248833, 10000.858573495, 151.57246907666666], [10003.256460254, 10000.647262600665, 151.572453818], [10003.292292917646, 10000.433191881762, 151.57240744294114], [10003.294170225881, 9999.565082764704, 151.57231678647057], [10003.070385321427, 9998.726710429999, 151.5722928735714], [10002.636924780769, 9997.975230383847, 151.57251680846156], [10002.022833014, 9997.361623413333, 151.57221883], [10001.270324762307, 9996.928998434614, 151.57242173461538], [10000.432645787858, 9996.706033060002, 151.57248251714287], [9999.565267393335, 9996.706937417777, 151.5722995361111], [9998.72808335091, 9996.931133226362, 151.57232804727272], [9997.977203743572, 9997.363958612856, 151.572380065], [9997.365728942143, 9997.978691664282, 151.57235608714288], [9996.931402103572, 9998.728589142858, 151.5722558157143], [9996.705542787333, 9999.565443869331, 151.57224934866665], [9996.704440150716, 10000.432576714285, 151.5725326542857], [9996.929543998334, 10001.27103265, 151.5725292638889], [9997.363204755386, 10002.02120515, 151.57220458999998], [9997.977003606667, 10002.633855913997, 151.57243448933332], [9998.72843960625, 10003.066585958126, 151.57242298125], [9999.565360932667, 10003.291790325335, 151.57230224533333]]

points47 = [[10000.431575435623, 10003.28438733375, 151.48999405249995], [10000.645533996429, 10003.249060579285, 151.48984527428573], [10000.751146214287, 10003.226131012856, 151.48999568500003], [10000.856645494288, 10003.199684977142, 151.48987906285717], [10001.06386259, 10003.136752639999, 151.4897024964286], [10001.266973951764, 10003.05998054647, 151.48976135235296], [10001.464456160833, 10002.970590154166, 151.48982493083332], [10001.655287096668, 10002.8683366575, 151.48989995166667], [10001.839462234613, 10002.753555059999, 151.48973905076923], [10002.015689200713, 10002.627339635, 151.48991938857145], [10002.183201000767, 10002.489485135382, 151.4899315469231], [10002.341568678, 10002.3410968, 151.48983561333333], [10002.489514027056, 10002.183238409412, 151.4898080264706], [10002.626911988464, 10002.01591014923, 151.4897437469231], [10002.753488553888, 10001.839511269443, 151.48984697222224], [10002.86791859286, 10001.65555984643, 151.48983219499996], [10002.97018726077, 10001.464592923847, 151.48977309], [10003.06013620846, 10001.266850700002, 151.48992919846154], [10003.136680053845, 10001.064166389231, 151.48974139846155], [10003.200079093078, 10000.85654123154, 151.48986346846152], [10003.249690553333, 10000.645463079167, 151.48986562000002], [10003.285387451999, 10000.431805994003, 151.48994649266666], [10003.286879460002, 9999.566658163889, 151.48986646833333], [10003.0637356625, 9998.730015321875, 151.48974990812502], [10002.631176263127, 9997.979391382501, 151.4899587625], [10002.0184486375, 9997.367230058751, 151.48981666499995], [10001.268057789284, 9996.935551745715, 151.48982020642862], [10000.431616374375, 9996.712943164375, 151.48993396812503], [9999.566309231177, 9996.71373659, 151.4899471517647], [9998.730551555454, 9996.937062133635, 151.4898861981818], [9997.982066936429, 9997.370278697144, 151.48981802785713], [9997.371899080666, 9997.982653871999, 151.489774576], [9996.938206563846, 9998.731006393846, 151.48987638076923], [9996.712478002668, 9999.566582867334, 151.489904786], [9996.711503077333, 10000.431819398667, 151.49001159733334], [9996.935944012142, 10001.267805140713, 151.48981911785717], [9997.369101856111, 10002.016851294444, 151.48993004666667], [9997.981268703752, 10002.627972664997, 151.489931105625], [9998.731195305, 10003.060008593573, 151.49003165214285], [9999.566064212142, 10003.283800567859, 151.48989650285714]]

points50 = [[10000.430190330666, 10003.277268044667, 151.4199839253333], [10000.643932220715, 10003.242021800714, 151.41993931285714], [10000.749697813748, 10003.219060108126, 151.420024871875], [10000.854534264, 10003.192725515333, 151.41991475533334], [10001.061336993751, 10003.129715888126, 151.419962883125], [10001.264087052943, 10003.053282022353, 151.42003676588234], [10001.460477173334, 10002.964078803334, 151.4198671975], [10001.651930002308, 10002.861725933844, 151.41992657076923], [10001.835204287143, 10002.74759820357, 151.41979762571427], [10002.011024603846, 10002.621403253845, 151.41992656923074], [10002.178255356923, 10002.48400161846, 151.42009559076922], [10002.336337089333, 10002.335704468003, 151.41996459933335], [10002.483870618824, 10002.178302595292, 151.4200690782353], [10002.620913933997, 10002.011501312001, 151.42000223866665], [10002.747048140001, 10001.835656687692, 151.41992070153844], [10002.861320990001, 10001.651961625, 151.42007337214287], [10002.963757149231, 10001.461057296154, 151.4200685946154], [10003.053083795834, 10001.264389644166, 151.42004776], [10003.129507413845, 10001.061834921538, 151.41991483307692], [10003.19286647154, 10000.854592376923, 151.41996882538461], [10003.242318114168, 10000.644208336666, 151.41991806083334], [10003.277976330588, 10000.430771117646, 151.41998470529413], [10003.279574729333, 9999.567322535335, 151.42014261800003], [10003.056864922308, 9998.732467386153, 151.42001460153844], [10002.625213297275, 9997.983743796365, 151.42001065272729], [10002.014214624545, 9997.373035605453, 151.42007168727275], [10001.264789352144, 9996.94216673714, 151.4199796392857], [10000.430220401333, 9996.720187520668, 151.42002461733333], [9999.567377427857, 9996.721063935713, 151.4200635642857], [9998.73350328143, 9996.944190418571, 151.41993604428572], [9997.986816645, 9997.37672644, 151.42009735142852], [9997.377583564166, 9997.987671376666, 151.4199256908333], [9996.944875837999, 9998.734011101, 151.419975282], [9996.71978015643, 9999.567384355003, 151.41986847], [9996.718745066155, 10000.430898540772, 151.41995356692308], [9996.94281864, 10001.265207291539, 151.42007681153848], [9997.37487857643, 10002.012306282142, 151.42001233785712], [9997.985907315715, 10002.622196538568, 151.42009843999998], [9998.73374352923, 10003.053140182306, 151.42008150615382], [9999.567365400666, 10003.276654021332, 151.420098876]]

points52 = [[10000.42925414643, 10003.268481442143, 151.35012817071427], [10000.642526798667, 10003.233092404, 151.35002034533332], [10000.747543468668, 10003.210549196667, 151.35016581066665], [10000.852337561248, 10003.184126258124, 151.3502225875], [10001.058885507775, 10003.121222879443, 151.35008070222221], [10001.260377447778, 10003.045257899443, 151.35017225555555], [10001.457347471998, 10002.955893134666, 151.35010986333333], [10001.647109064003, 10002.854261526, 151.35015869199998], [10001.830309925237, 10002.740089258095, 151.35018630190476], [10002.005600483999, 10002.614298629334, 151.35007019066668], [10002.17232595143, 10002.477298565002, 151.35015978285713], [10002.329198785714, 10002.330067839288, 151.35006169071428], [10002.476705284707, 10002.172797721765, 151.3502619141176], [10002.613806579999, 10002.005737828667, 151.35010274133336], [10002.74004906923, 10001.830017685383, 151.3499814553846], [10002.853502342143, 10001.647469060716, 151.35008239785716], [10002.955621599167, 10001.4569555025, 151.35022481083334], [10003.044692555553, 10001.26088010611, 151.35014343388886], [10003.121052900833, 10001.059046419166, 151.3500696825], [10003.183989597692, 10000.852705073075, 151.35006479153844], [10003.233483113077, 10000.642266524615, 151.35008591923076], [10003.269010603335, 10000.429548916669, 151.35017394916665], [10003.270615613847, 9999.568456463077, 151.35005657461537], [10003.048832995, 9998.736353057144, 151.35018921], [10002.618099689169, 9997.9890083075, 151.35024388500003], [10002.008840084998, 9997.380118470714, 151.35021863642856], [10001.261511161538, 9996.950465773074, 151.35030423846155], [10000.429336747498, 9996.72899223875, 151.35024356812502], [9999.568412431876, 9996.73013140125, 151.350070951875], [9998.737079749999, 9996.9524931725, 151.35028203166667], [9997.992471808822, 9997.383901975883, 151.35016407705882], [9997.384743853845, 9997.99323467154, 151.3501046992308], [9996.95307212357, 9998.73750387, 151.3501892085714], [9996.728561305332, 9999.568353200664, 151.3500488286667], [9996.727416625385, 10000.42950292077, 151.35011056846153], [9996.950886188748, 10001.261790558752, 151.35025310562503], [9997.382199695714, 10002.00710015643, 151.35015651142857], [9997.991591982145, 10002.615107144287, 151.35019138857143], [9998.7374103275, 10003.044801175, 151.35020701083332], [9999.568257334167, 10003.267726996666, 151.35010401333332]]

points56 = [[10000.427857576844, 10003.256086399999, 151.26748817947367], [10000.639856656668, 10003.22109801, 151.26772867866663], [10000.744874153685, 10003.198354794737, 151.26745284315788], [10000.849242761178, 10003.172100684704, 151.26755119705882], [10001.05460499857, 10003.109557014284, 151.26765659857145], [10001.256005067144, 10003.033696685714, 151.2675857542857], [10001.4516937625, 10002.94481202, 151.26775105750002], [10001.640901572631, 10002.843423027894, 151.26751628631575], [10001.82362006154, 10002.729663299231, 151.26767437307694], [10001.998145315387, 10002.604258885385, 151.26739854153848], [10002.164032949999, 10002.468034520625, 151.26754760625], [10002.32073827611, 10002.321038950002, 151.2675959277778], [10002.467463875333, 10002.164458449335, 151.26760152133332], [10002.604021644667, 10001.997993271334, 151.26775818], [10002.729275989997, 10001.823519, 151.26762797133333], [10002.842845311538, 10001.64084261923, 151.26763564], [10002.944208757855, 10001.45153657, 151.26749747000002], [10003.033168291999, 10001.255972707, 151.267703248], [10003.109265343333, 10001.054704078666, 151.26766866199995], [10003.171785208462, 10000.849577375386, 151.26744549153847], [10003.221048610001, 10000.640011804, 151.2674957273333], [10003.256548406154, 10000.428182152307, 151.26753939076923], [10003.25837216077, 9999.569552073848, 151.2676567676923], [10003.03702047857, 9998.740135072858, 151.26774052071428], [10002.608359491764, 9997.996399921765, 151.2676822435294], [10002.000997171668, 9997.389530103888, 151.26768832833332], [10001.256903663334, 9996.961935664667, 151.26762288533334], [10000.427368262664, 9996.741184821998, 151.26751810800002], [9999.570113201, 9996.742309786001, 151.26756286900002], [9998.741656467693, 9996.964046000769, 151.2676462030769], [9997.999668987857, 9997.394265600717, 151.26747894285714], [9997.394860176155, 9998.000749149232, 151.2676508976923], [9996.964534585453, 9998.742194554547, 151.26763361272728], [9996.740693346665, 9999.570418133333, 151.2675964373333], [9996.739556440665, 10000.427909999333, 151.26759948866666], [9996.96222082, 10001.25690872182, 151.26755593090908], [9997.391745845001, 10001.9991015, 151.26751327416665], [9997.99884011, 10002.604849082858, 151.26749093357145], [9998.741920991873, 10003.033183946874, 151.26747322125001], [9999.570088427858, 10003.255561251428, 151.26740809857142]]

el29 = [0.229, 0.226, 0.187, 0.197, 0.197, 0.212, 0.199, 0.187, 0.19, 0.169, 0.177, 0.167, 0.187, 0.177, 0.197, 0.215,

0.222, 0.222, 0.227, 0.241, 0.257, 0.268, 0.326, 0.361, 0.397, 0.407, 0.387, 0.355, 0.329, 0.339, 0.305, 0.287,

0.325, 0.36, 0.387, 0.362, 0.338, 0.307, 0.285, 0.249]

el32 = [0.164, 0.152, 0.124, 0.205, 0.162, 0.178, 0.155, 0.162, 0.172, 0.142, 0.135, 0.143, 0.143, 0.157, 0.172, 0.192,

0.186, 0.188, 0.16, 0.174, 0.192, 0.207, 0.265, 0.305, 0.335, 0.35, 0.327, 0.295, 0.27, 0.327, 0.311, 0.232,

0.272, 0.31, 0.332, 0.309, 0.282, 0.244, 0.213, 0.205]

el35 = [0.169, 0.157, 0.119, 0.178, 0.169, 0.169, 0.163, 0.179, 0.151, 0.131, 0.159, 0.146, 0.142, 0.157, 0.174, 0.181,

0.179, 0.187, 0.139, 0.154, 0.174, 0.189, 0.249, 0.299, 0.328, 0.339, 0.311, 0.287, 0.26, 0.329, 0.314, 0.229,

0.269, 0.317, 0.329, 0.307, 0.27, 0.229, 0.199, 0.189]

el38 = [0.186, 0.197, 0.151, 0.201, 0.201, 0.19, 0.208, 0.186, 0.175, 0.156, 0.156, 0.166, 0.176, 0.166, 0.171, 0.186,

0.204, 0.203, 0.166, 0.186, 0.205, 0.216, 0.281, 0.326, 0.362, 0.376, 0.348, 0.316, 0.298, 0.354, 0.348, 0.256,

0.301, 0.348, 0.364, 0.336, 0.298, 0.258, 0.231, 0.22]

el41 = [0.162, 0.145, 0.136, 0.194, 0.205, 0.177, 0.161, 0.182, 0.162, 0.157, 0.138, 0.142, 0.152, 0.119, 0.133, 0.164,

0.162, 0.162, 0.142, 0.158, 0.174, 0.194, 0.262, 0.309, 0.358, 0.357, 0.328, 0.293, 0.279, 0.312, 0.337, 0.234,

0.278, 0.331, 0.347, 0.32, 0.281, 0.229, 0.199, 0.202]

el44 = [0.157, 0.148, 0.146, 0.148, 0.162, 0.132, 0.143, 0.125, 0.152, 0.119, 0.125, 0.107, 0.127, 0.117, 0.102, 0.145,

0.127, 0.137, 0.141, 0.156, 0.174, 0.197, 0.26, 0.312, 0.352, 0.359, 0.33, 0.297, 0.283, 0.308, 0.317, 0.24,

0.277, 0.337, 0.35, 0.322, 0.277, 0.228, 0.173, 0.157]

el47 = [0.203, 0.196, 0.192, 0.188, 0.176, 0.17, 0.162, 0.152, 0.146, 0.139, 0.138, 0.131, 0.131, 0.132, 0.139, 0.151,

0.157, 0.169, 0.181, 0.196, 0.212, 0.234, 0.304, 0.361, 0.399, 0.409, 0.376, 0.339, 0.326, 0.361, 0.321, 0.273,

0.321, 0.379, 0.401, 0.366, 0.321, 0.27, 0.215, 0.202]

el50 = [0.218, 0.209, 0.204, 0.201, 0.194, 0.184, 0.176, 0.167, 0.16, 0.166, 0.149, 0.14, 0.142, 0.144, 0.146, 0.154,

0.169, 0.174, 0.188, 0.204, 0.222, 0.244, 0.318, 0.374, 0.418, 0.424, 0.39, 0.351, 0.334, 0.352, 0.317, 0.28,

0.334, 0.394, 0.414, 0.383, 0.334, 0.279, 0.234, 0.214]

el52 = [0.240, 0.231, 0.227, 0.222, 0.212, 0.204, 0.195, 0.188, 0.177, 0.168, 0.159, 0.156, 0.158, 0.158, 0.161, 0.168,

0.177, 0.194, 0.202, 0.219, 0.235, 0.259, 0.336, 0.393, 0.435, 0.442, 0.408, 0.372, 0.352, 0.373, 0.328, 0.292,

0.354, 0.415, 0.437, 0.402, 0.35, 0.292, 0.239, 0.235]

el56 = [0.259, 0.251, 0.244, 0.24, 0.231, 0.222, 0.212, 0.204, 0.192, 0.179, 0.173, 0.17, 0.17, 0.17, 0.171, 0.178,

0.187, 0.198, 0.211, 0.237, 0.248, 0.272, 0.351, 0.41, 0.453, 0.461, 0.427, 0.384, 0.371, 0.384, 0.342, 0.301,

0.37, 0.433, 0.457, 0.419, 0.364, 0.303, 0.252, 0.251]

new = []

for p in el29:

new += [p \* .0254]

el29 = list(new)

new = []

for p in el32:

new += [p \* .0254]

el32 = list(new)

new = []

for p in el35:

new += [p \* .0254]

el35 = list(new)

new = []

for p in el38:

new += [p \* .0254]

el38 = list(new)

new = []

for p in el41:

new += [p \* .0254]

el41 = list(new)

new = []

for p in el44:

new += [p \* .0254]

el44 = list(new)

new = []

for p in el47:

new += [p \* .0254]

el47 = list(new)

new = []

for p in el50:

new += [p \* .0254]

el50 = list(new)

new = []

for p in el52:

new += [p \* .0254]

el52 = list(new)

new = []

for p in el56:

new += [p \* .0254]

el56 = list(new)

new = []

points = points29 + points32 + points35 + points38 + points41 + points44 + points47 + points50 + points52 + points56

dist = el29 + el32 + el35 + el38 + el41 + el44 + el47 + el50 + el52 + el56

# starting parameters

xc = 10000.000615397612

yc = 10000.00040379931

zc = 151.82316465831497

rc = 3.3240036718929082

alpha = 0.0007 # learning rate

mse = 10

while mse > 0.00047707:

x\_change = 0

y\_change = 0

z\_change = 0

r\_change = 0

mse = 0

for i in range(0, len(points)):

mse += (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \*\* 2

x\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

(1 / 2) \* (1 / math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)))) \* (

2 \* (xc - points[i][0]))

y\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

(1 / 2) \* (1 / math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)))) \* (

2 \* (yc - points[i][1]))

z\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

-0.5 \* (1 / (math.sqrt(rc \*\* 2 - (zc - points[i][2]))))) \* (-2 \* (zc - points[i][2]))

r\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

-0.5 \* (1 / (math.sqrt(rc \*\* 2 - (zc - points[i][2]))))) \* (2 \* rc)

mse = math.sqrt(mse / len(points))

print(mse)

rmse = (1 / mse) \* 0.5

x\_change = (x\_change / len(points)) \* rmse

y\_change = (y\_change / len(points)) \* rmse

z\_change = (z\_change / len(points)) \* rmse

r\_change = (r\_change / len(points)) \* rmse

xc = xc - alpha \* x\_change

yc = yc - alpha \* y\_change

zc = zc - alpha \* z\_change

rc = rc - alpha \* r\_change

print(xc, yc, zc, rc)

# xc = 10000.000615212366

# yc = 10000.000403627007

# zc = 151.82313134652927

# rc = 3.3244697186758834

# xc = 10000.000639526275

# yc = 10000.00042639083

# zc = 151.82326315456376

# rc = 3.3239622768323454

# xc = 10000.0007

# yc = 10000.0005

# zc = 151.826

# rc = 3.324

for i in range(0, len(points)):

mse += (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \*\* 2

mse = math.sqrt(mse / len(points))

print(mse)

y = [1.0, 1.25, 1.375, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, 3.25, 3.5, 3.75, 4.0, 4.25, 4.5, 4.75, 5.0, 5.25, 5.5,

5.75, 6.0, 7.0, 8.0, 9.0, 10.0, 11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0, 19.0, 20.0, 21.0, 22.0, 23.0, 24.0]

x29 = []

for i in range(0, len(points29)):

x29 += [1 \* (math.sqrt(((xc - points29[i][0]) \*\* 2) + ((yc - points29[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points29[i][2]) \*\* 2)))]

f29 = plt.figure()

plt.title("29\" elevation")

plt.plot(y, x29, label="point cloud")

plt.plot(y, el29, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("29(range).png")

plt.close()

x32 = []

for i in range(0, len(points29)):

x32 += [1 \* (math.sqrt(((xc - points32[i][0]) \*\* 2) + ((yc - points32[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points32[i][2]) \*\* 2)))]

f32 = plt.figure()

plt.title("32\" elevation")

plt.plot(y, x32, label="point cloud")

plt.plot(y, el32, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("32(range).png")

plt.close()

x35 = []

for i in range(0, len(points29)):

x35 += [1 \* (math.sqrt(((xc - points35[i][0]) \*\* 2) + ((yc - points35[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points35[i][2]) \*\* 2)))]

f35 = plt.figure()

plt.title("35\" elevation")

plt.plot(y, x35, label="point cloud")

plt.plot(y, el35, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("35(range).png")

plt.close()

x38 = []

for i in range(0, len(points29)):

x38 += [1 \* (math.sqrt(((xc - points38[i][0]) \*\* 2) + ((yc - points38[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points38[i][2]) \*\* 2)))]

f38 = plt.figure()

plt.title("38\" elevation")

plt.plot(y, x38, label="point cloud")

plt.plot(y, el38, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("38(range).png")

plt.close()

x41 = []

for i in range(0, len(points29)):

x41 += [1 \* (math.sqrt(((xc - points41[i][0]) \*\* 2) + ((yc - points41[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points41[i][2]) \*\* 2)))]

f41 = plt.figure()

plt.title("41\" elevation")

plt.plot(y, x41, label="point cloud")

plt.plot(y, el41, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("41(range).png")

plt.close()

x44 = []

for i in range(0, len(points29)):

x44 += [1 \* (math.sqrt(((xc - points44[i][0]) \*\* 2) + ((yc - points44[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points44[i][2]) \*\* 2)))]

f44 = plt.figure()

plt.title("44\" elevation")

plt.plot(y, x44, label="point cloud")

plt.plot(y, el44, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("44(range).png")

plt.close()

x47 = []

for i in range(0, len(points29)):

x47 += [1 \* (math.sqrt(((xc - points47[i][0]) \*\* 2) + ((yc - points47[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points47[i][2]) \*\* 2)))]

f47 = plt.figure()

plt.title("47\" elevation")

plt.plot(y, x47, label="point cloud")

plt.plot(y, el47, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("47(range).png")

plt.close()

x50 = []

for i in range(0, len(points29)):

x50 += [1 \* (math.sqrt(((xc - points50[i][0]) \*\* 2) + ((yc - points50[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points50[i][2]) \*\* 2)))]

f50 = plt.figure()

plt.title("50\" elevation")

plt.plot(y, x50, label="point cloud")

plt.plot(y, el50, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("50(range).png")

plt.close()

x52 = []

for i in range(0, len(points29)):

x52 += [1 \* (math.sqrt(((xc - points52[i][0]) \*\* 2) + ((yc - points52[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points52[i][2]) \*\* 2)))]

f52 = plt.figure()

plt.title("52\" elevation")

plt.plot(y, x52, label="point cloud")

plt.plot(y, el52, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("52(range).png")

plt.close()

x56 = []

for i in range(0, len(points29)):

x56 += [1 \* (math.sqrt(((xc - points56[i][0]) \*\* 2) + ((yc - points56[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points56[i][2]) \*\* 2)))]

f56 = plt.figure()

plt.title("56\" elevation")

plt.plot(y, x56, label="point cloud")

plt.plot(y, el56, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("56(range).png")

plt.close()

"""

x = []

y = []

for point in points:

x += [point[0]]

y += [point[1]]

fig, ax = plt.subplots()

circle2 = plt.Circle((xc, yc), rc, color='b', fill=True)

ax.plot(x, y, 'o')

ax.add\_artist(circle2)

fig.savefig('plotcircles2.png')

"""